

CORRIGÉ

3. Exercices équivalents

1. $\frac{3n + \ln(n)}{3^n + 2^n} \sim \frac{3n}{3^n}$
2. $n \sin\left(\frac{1}{n \ln n}\right) \sim \frac{1}{\ln n}$
3. $\sqrt{n^2 - n + 1} - n \sim -\frac{1}{2}$
4. $\ln\left(\frac{n^2 + 1}{n^2 - 1}\right) \sim \frac{2}{n^2}$
5. $\cos(1/n) - \frac{n+1}{n} \sim -\frac{1}{n}$
6. $\exp(n + n^2) \sim \exp(n + n^2)$
7. $\frac{n! + n \cos(n)}{n + n \ln n} \sim \frac{(n-1)!}{\ln n}$
8. $\tan\left(\frac{n^2 \pi + 1}{n}\right) \sim \frac{1}{n}$
9. $\ln(\cos(1/n^2)) \sim -\frac{1}{2n^4}$
10. $\sqrt[3]{n^4 + 3n - 1} \sim n^{4/3}$
11. $\frac{1}{1 - \sin(1/n)} - 1 \sim \frac{1}{n}$
12. $\left(\frac{n^2 + n + 1}{n^2 - n + 1}\right)^n \sim e^2$

4. Exercices développements limités

1. $\sin x + x \cos x = x - \frac{2}{3}x^3 + o(x^3)$
2. $e^x \ln(1+x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + o(x^3)$
3. $\sqrt{1+x^2} + \cos^2 x = 2 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + o(x^4)$
4. $\ln(1 + \sin x) = x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + o(x^3)$
5. $\frac{\cos x}{1 + \sin x} = 1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + o(x^3)$
6. $\sqrt{1+e^x} = \sqrt{2} + \frac{1}{2\sqrt{2}}x + \frac{3}{16\sqrt{2}}x^2 + \frac{7}{1024\sqrt{2}}x^3 + o(x^3)$.
7. $\frac{1}{x} - \frac{1}{\sin x} = -\frac{1}{6}x - \frac{7}{360}x^3 + o(x^3)$
8. $\cos(x) = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{\pi}{3}) - \frac{1}{4}(x - \frac{\pi}{3})^2 + \frac{\sqrt{3}}{12}(x - \frac{\pi}{3})^3 + o((x - \frac{\pi}{3})^3)$.
9. $\ln(\ln(x)) = \frac{1}{e}(x - e) - \frac{1}{e^2}(x - e)^2 + \frac{7}{6e^3}(x - e)^3 + o((x - e)^3)$
10. $\arctan x + \frac{1}{1+x^2} = 1 + x - x^2 - \frac{1}{3}x^3 + o(x^3)$
11. $(\cos x)^{\sin x} = 1 - \frac{1}{2}x^3 + o(x^3)$
12. $\int_0^x \frac{dt}{1+t} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + o(x^3)$.